RELATIVISTIC QUANTUM MECHANICS

Thursday 06-04-2014, 09.00-12.00

Write your name and student number on **all** sheets. The total number of points is 90. You can earn 5 points for each subquestion, except for 2b, 2d, 3a and 3b (10 points) and 4c (15 points).

Use conventions with $\hbar = c = 1$.

PROBLEM 1: HARMONIC OSCILLATOR

Comment in one or two lines on the difference or similarity between the quantum-mechanical system of the harmonic oscillator and a free field theory regarding:

a. the structure of ladder operators,

b. the number of ladders,

c. the frequency and energy spacing of ladders.

PROBLEM 2: FOURIER TRANSFORMATIONS

Throughout this problem we will use the Heisenberg picture for operators.

a. What is the decomposition into ladder operators of the operator for a relativistic real scalar field ϕ and its conjugate momentum π?

b. Derive the inverse relations for the ladder operators in terms of the field ϕ and its momentum π.

c. What is the decomposition into ladder operators of the operator for a non-relativistic complex scalar field ψ and its conjugate momentum π?

d. Derive the inverse relations for the ladder operators in terms of the field ψ and its momentum π.

PROBLEM 3: MAJORANA CONDITION

Instead of the Weyl condition that selects a chirality component of the Dirac spinor, one could also impose the Majorana condition. It takes the simplest form when using the Majorana representation for the gamma-matrices, given by

$$\gamma^0 = \begin{pmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} i\sigma_3 & 0 \\ 0 & i\sigma_3 \end{pmatrix}, \quad \gamma^0 = \begin{pmatrix} 0 & -\sigma_2 \\ \sigma_2 & 0 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} -i\sigma_1 & 0 \\ 0 & -i\sigma_1 \end{pmatrix},$$

with the usual expressions for the Pauli matrices σ_i. Using this representation, the Majorana condition on Dirac spinors is given by

$$\psi = \psi^* \tag{1}$$

a. Is this condition compatible with the Lorentz symmetry - in other words, how does this condition transform under a Lorentz transformation? Proof your answer.

b. Is this condition compatible with the massive Dirac equation - in other words, how does the massive Dirac operator act on a Majorana spinor? Proof your answer.

PROBLEM 4: SPINOR REPRESENTATION

a. What is the Lorentz transformation of a spinor ψ, and of its complex conjugate $\psi^\dagger$?

b. How does the combination $\bar{\psi}\psi$ transform under Lorentz transformations? Proof your answer.

c. How does the combination $\bar{\psi}\gamma^\mu\psi$ transform under Lorentz transformations? Proof your answer.